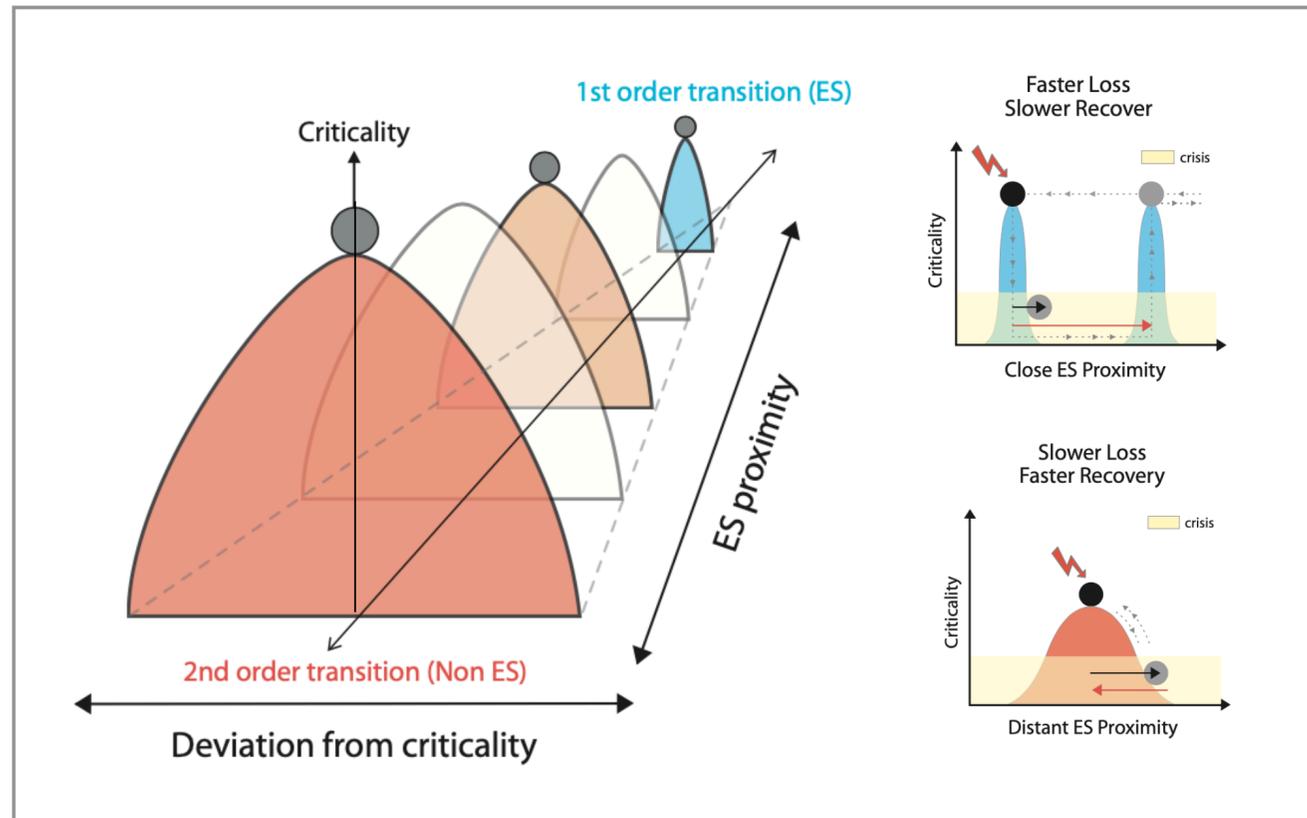


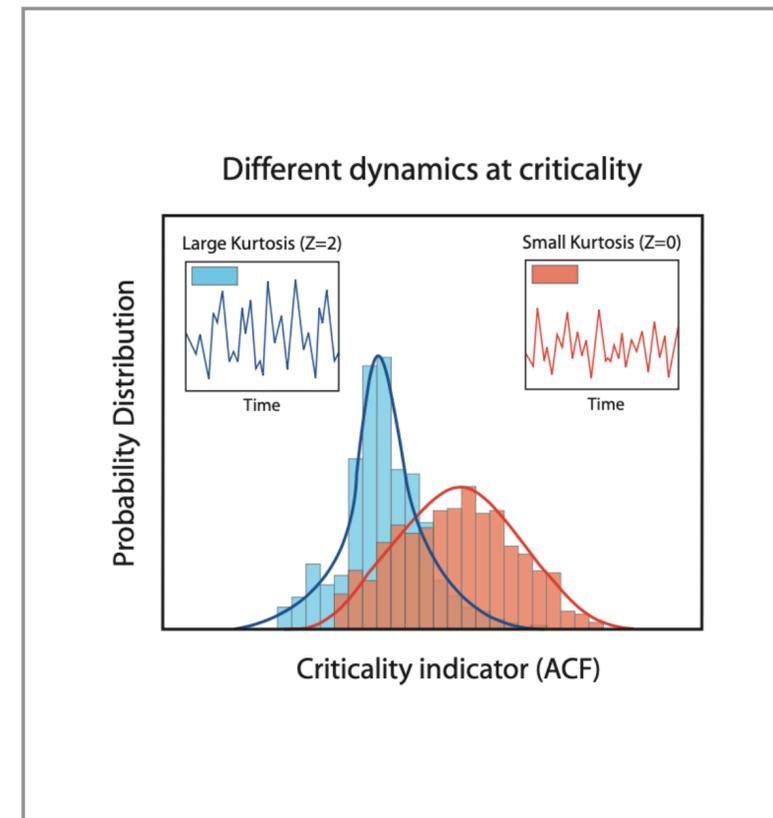
Schematic

Key Question: Can we infer a network's proximity to a first-order phase transition using time series signals, and use it to predict early or prolonged criticality loss and recovery under strong perturbations?

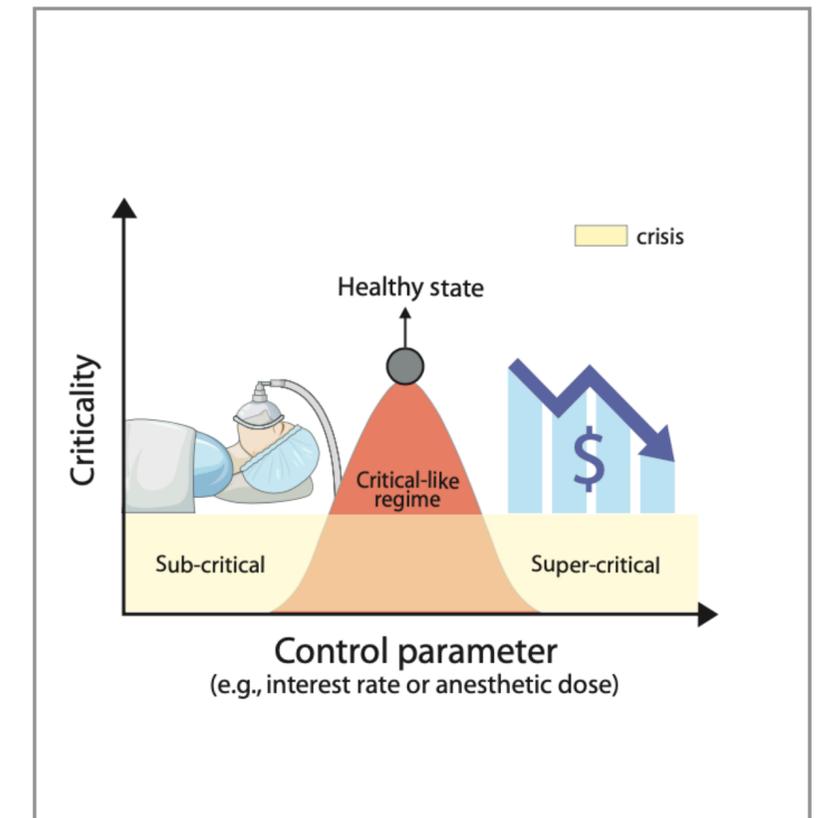


Patterns of brain criticality loss and recovery under strong external perturbation may be significantly influenced by **the network's phase transition type (first- or second-order).**

Explosive synchronization (ES) represents a first-order phase transition in complex networks. We propose that proximity to ES could determine early or prolonged criticality loss and recovery during external perturbations, such as neuronal or economic crises.

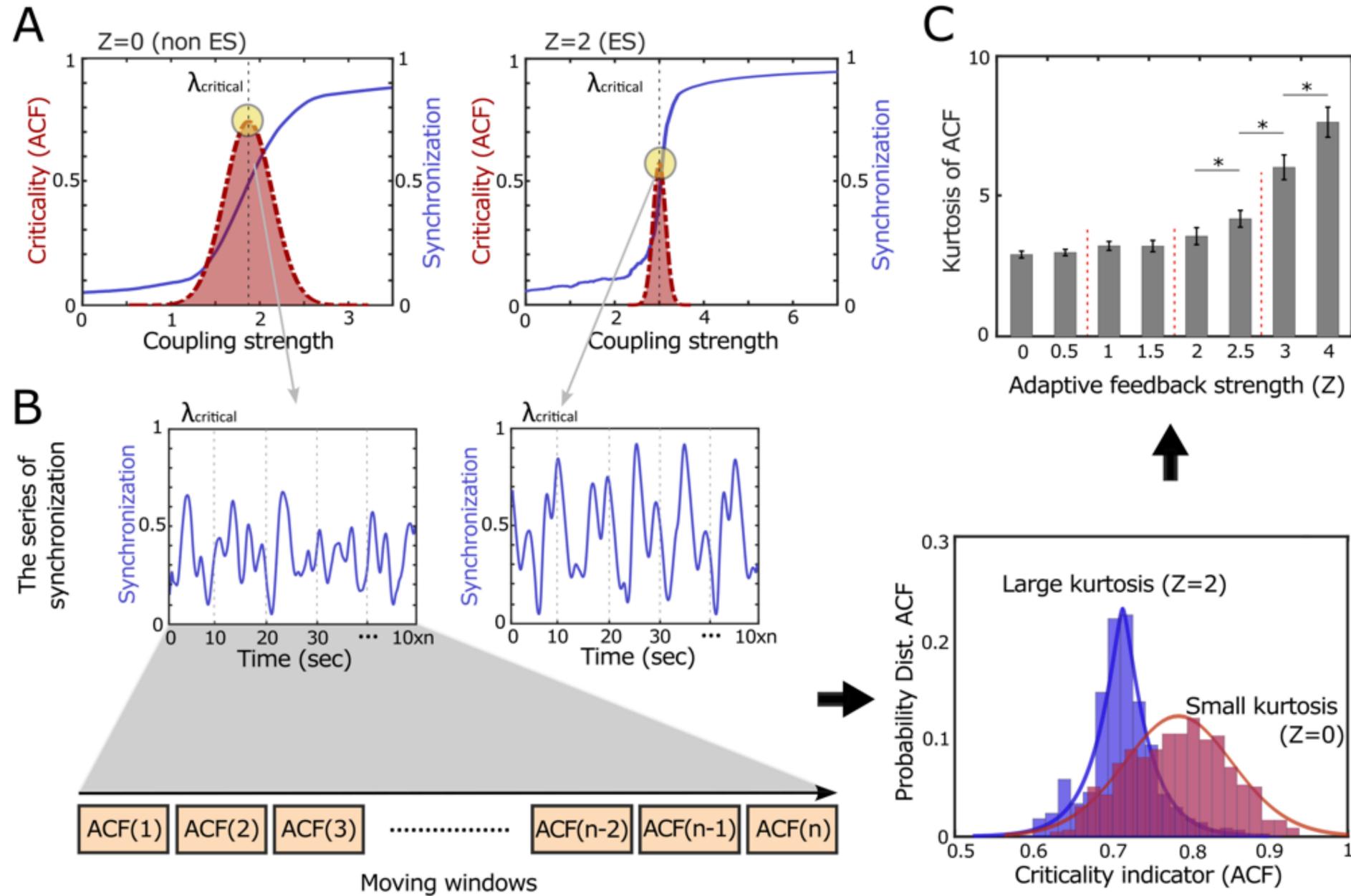


Our computational model reveals **distinct temporal dynamics** at critical points for networks with 'close' versus 'distant' proximities to ES.



Use empirical time series data to **infer** the ES proximity of brain and financial networks, **predicting early or prolonged** consciousness loss and recovery, as well as market collapse and recovery during neuronal and economic crises.

Computational Model 1:



Distinct network dynamics at critical points with varying explosive synchronization (ES) proximity.

Figure 2. (A) Networks with close ($Z=2$) and distant ($Z=0$) ES proximities exhibit distinctive network dynamics near their critical points. The network with closer ES proximity ($Z=2$) undergoes a steeper phase transition near its critical point than the network with distant ES proximity ($Z=0$). Critical points were identified based on the maximal autocorrelation functions (ACF) of instantaneous order parameters (global phase coherence) (indicated by circles). The red dotted cover indicates the ACF values at each coupling strength. (B) A moving window technique was employed to analyze the variability of ACF values, distinguishing close and distant ES proximities. (C) The ACF distribution of a close ES proximity ($Z=2$) has a larger kurtosis compared to that of the distant ES proximity network ($Z=0$). The distinct ACF distributions, that is, different instantaneous order parameter fluctuations at critical points, show the potential to estimate the ES proximity with time series data of a complex dynamical network.

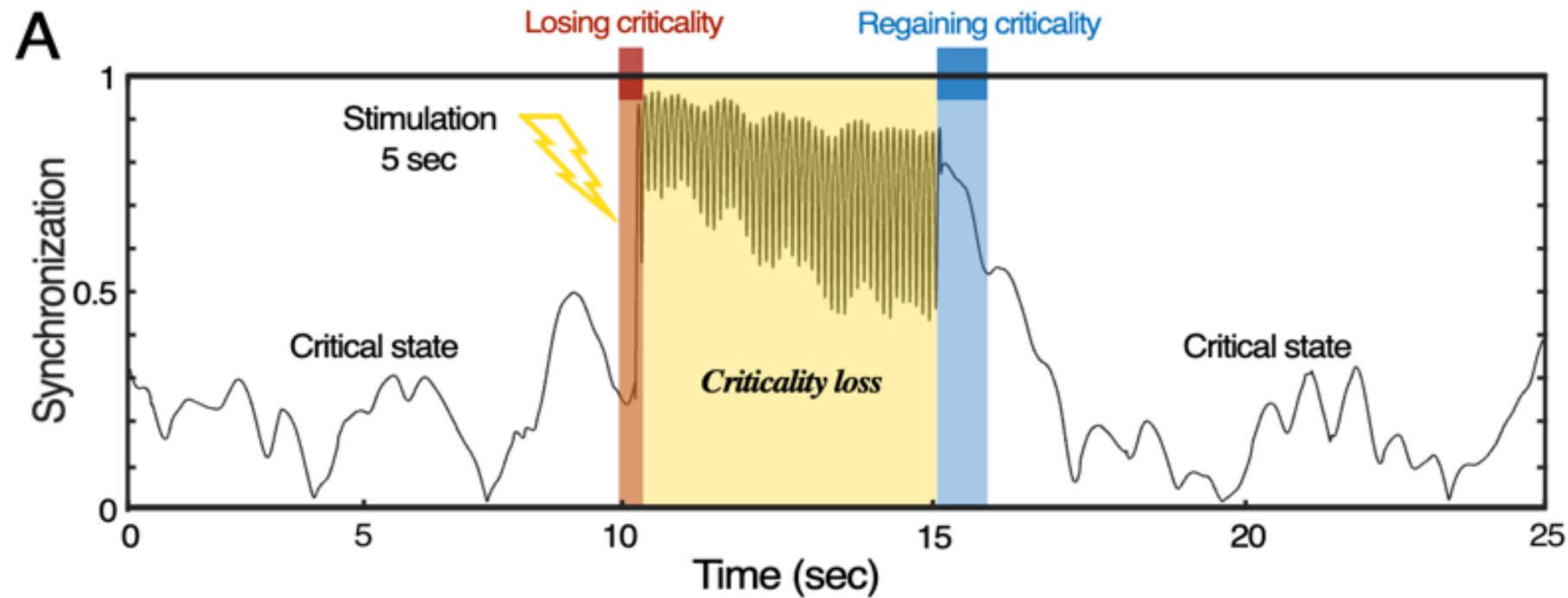
$$\dot{r}_j(t) = \{\lambda_j - |r_j(t)|^2\} r_j(t) + S \sum_{k=1}^N A_{jk} r_k \cos(\theta_k(t - \tau_{jk}) - \theta_j(t)) + u(t),$$

$$\dot{\theta}_j(t) = \omega_j + R_j^Z S \sum_{k=1}^N A_{jk} \frac{r_k}{r_j} \sin(\theta_k(t - \tau_{jk}) - \theta_j(t)), \quad j = 1, 2, \dots, N.$$

$$u(t) = \{p, t_1 < t < t_2, 0,$$

A modified Stuart-Landau Model

Computational Model 2:



The influence of ES proximity on the critical state loss and recovery under external perturbation.

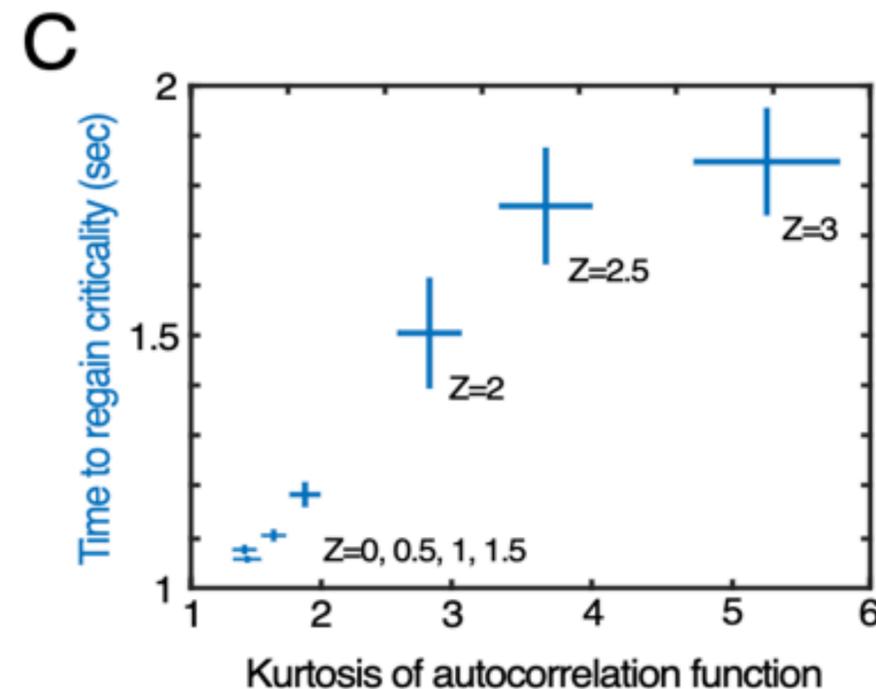
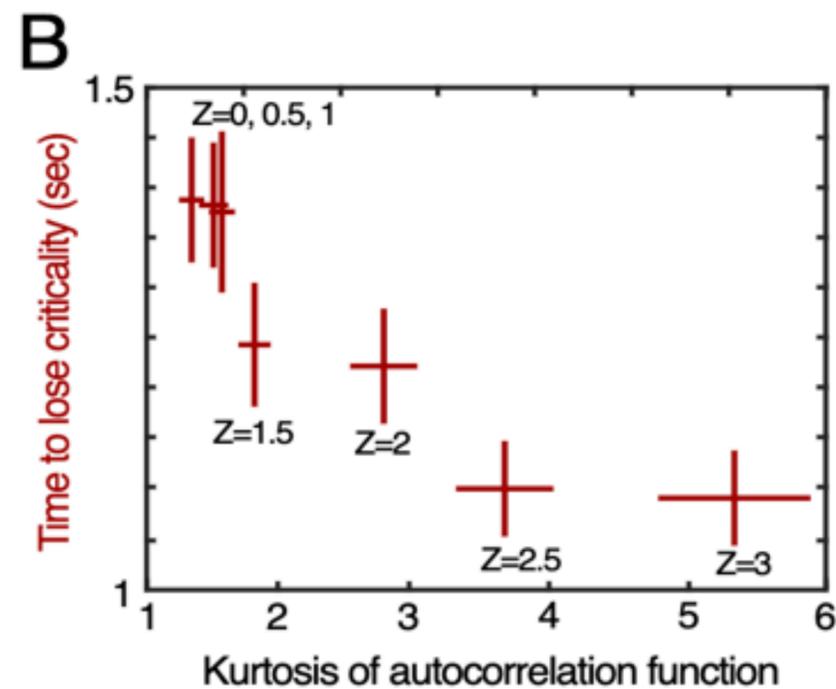
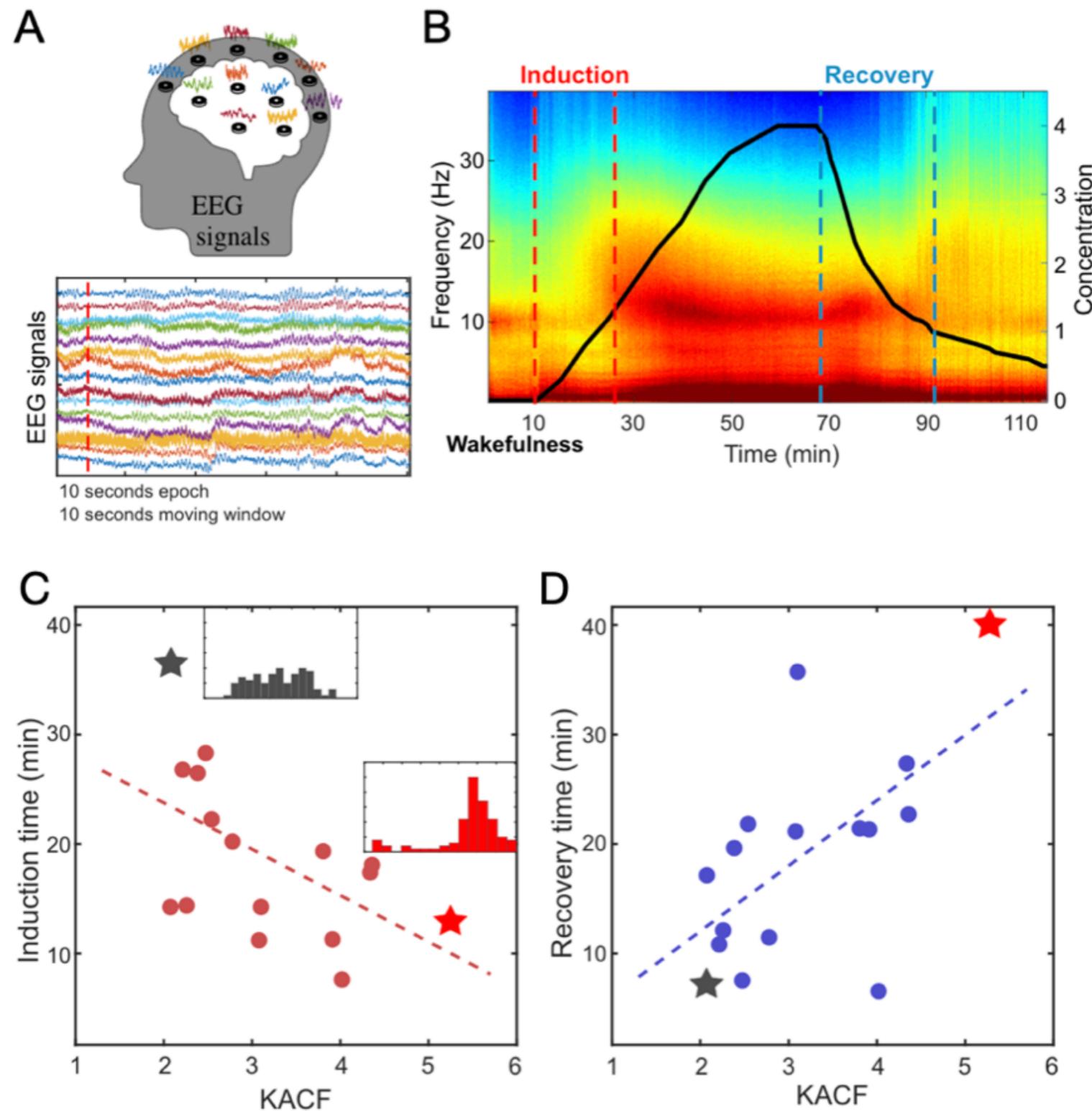


Figure 3.(A) a network with close ES proximity ($Z=2$) presents characteristic critical state loss and recovery patterns. Under external perturbation, the network loses and regains its baseline critical state. The times to lose (red zone) and regain (blue zone) the baseline states were determined by the times when the network's ACF values cross over and return to within three standard deviations of the ACF values in the baseline state. (B) The kurtoses of baseline ACF distributions are negatively correlated with the times to baseline critical state loss, indicating that networks with closer ES proximity are more prone to faster critical state loss. (C) Conversely, the kurtoses of baseline ACF distribution are positively correlated with the recovery time. A network with closer ES proximity exhibits slower critical state recovery. Error bars indicate standard deviations of 100 simulations.

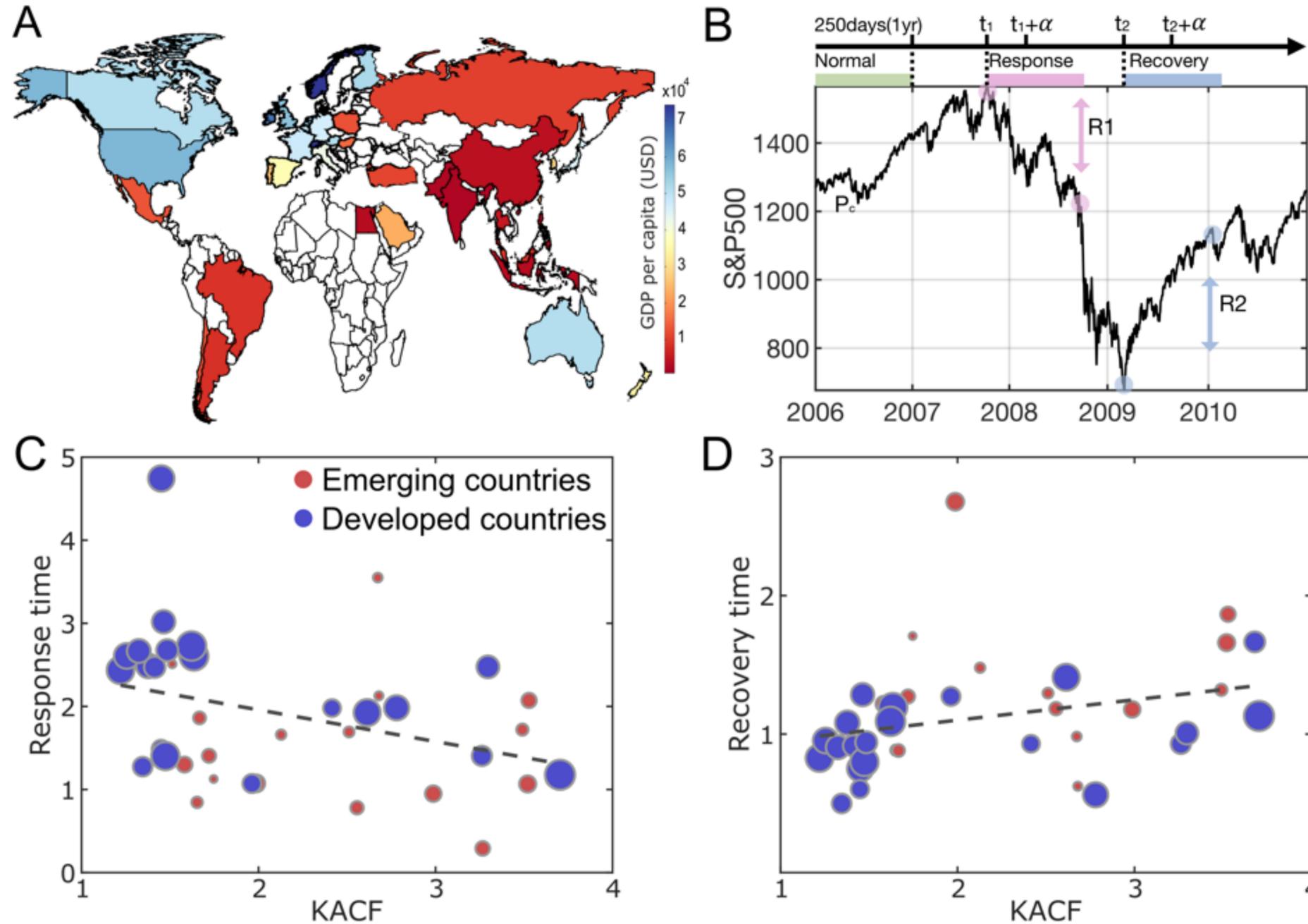
Human Brain Networks during Anesthesia



The ES proximity of human EEG in a conscious resting state significantly correlates with the induction time and the recovery time in general anesthesia.

Figure 4. (A) Thirty-two channel EEGs during baseline consciousness, induction of anesthesia, unconsciousness, and recovery states were analyzed to test the relationship between ES proximity in conscious brains and fast/slow state transitions induced by the general anesthetic propofol. (B) The spectrogram shows a significant change in the spectral content of the EEG along with state transitions in anesthesia. The anesthetic induction and recovery times were defined by the time intervals between the injection of propofol and the loss of responsiveness (red dotted lines) to a verbal command (induction) and between the end of injection and the recovery of response (blue dotted lines) to a verbal command (recovery). The solid line indicates the modeled effect-site concentration of propofol in the volunteer's brain. The kurtosis of ACF calculated with the baseline EEG shows a significant negative correlation with the induction times (C) for 16 subjects. Conversely, the kurtosis of the ACF of the baseline EEG positively correlates with the recovery time (D).

Stock Market Networks during Economic Crisis



The ES proximities of stock market networks significantly correlate with the rates of market collapse and recovery during the 2008 economic crisis.

Fig 5. (A) The thirty-nine countries analyzed in this study are mapped according to their 2006 GDP per capita (USD), with colors ranging from red to blue. (B) The market price of the S&P 500 index underwent a dramatic price change in the 2008-2009 economic crisis, which created natural epochs of baseline (pre-crisis), response (intra-crisis), and recovery (post-crisis) periods. The ES proximity of a stock market network was calculated in the baseline period, and the response and recovery times were calculated with the market collapse (R1) and recovery (R2) rates. (C) and (D) The kurtosis of ACF in the baseline period is negatively correlated with the response time ($\rho = -0.40$, $p < 0.001$), and positively correlated with the recovery time ($\rho = 0.49$, $p < 0.001$). Blue and red circles represent developed and emerging countries, respectively, and the marker sizes are scaled by the country's GDP.

Conclusion

Our computational model demonstrates a relationship between proximity to Explosive Synchronization (ES) and the trajectories of criticality loss and recovery. Networks closer to ES exhibit faster loss of critical state and slower recovery under strong perturbations.

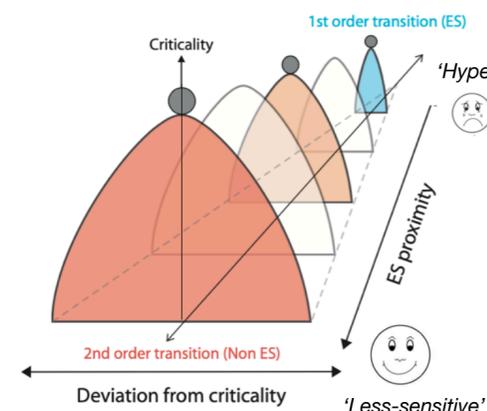
We validated these relationships by examining state transition in the human brain and financial networks during neuronal and economic crises.

Currently, we are applying this relationship to modulate prolonged recovery to normal recovery from anesthesia and coma-induced unconsciousness and to reduce hypersensitivity in chronic pain conditions such as fibromyalgia and sickle cell disease.

Current projects

(NIH R01 Project) Modulating Chronic Pain Severity

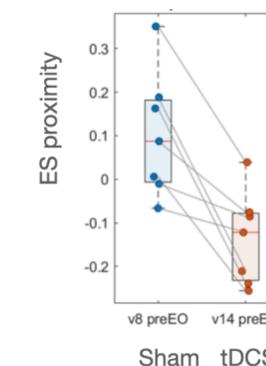
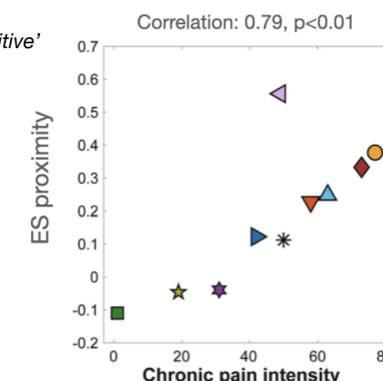
Modulating chronic pain brain from 'close' to 'distant' ES proximity



Fibromyalgia

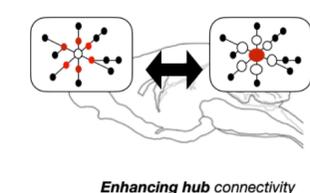


tDCS

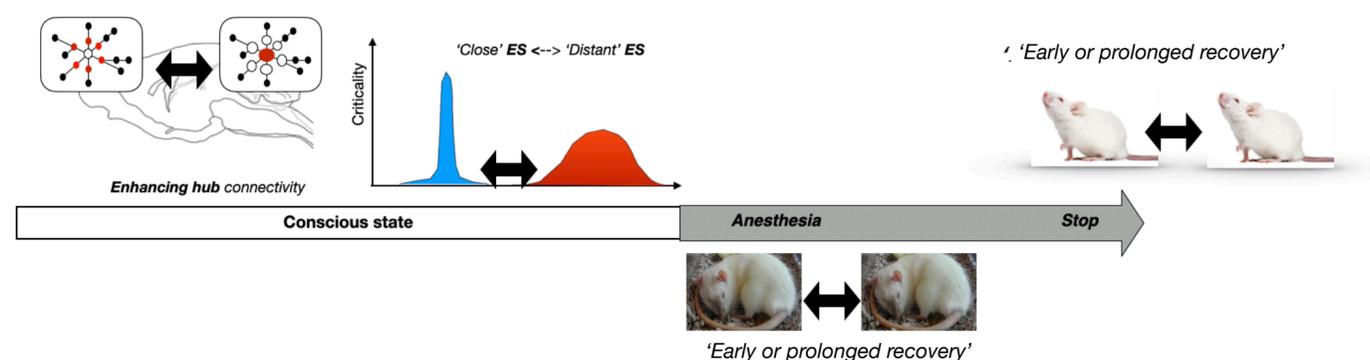


(NIH R21 Project) Modulating State Transition Trajectory

Brain Network Modulation



State Transition Trajectory Modulation



Robustness Test

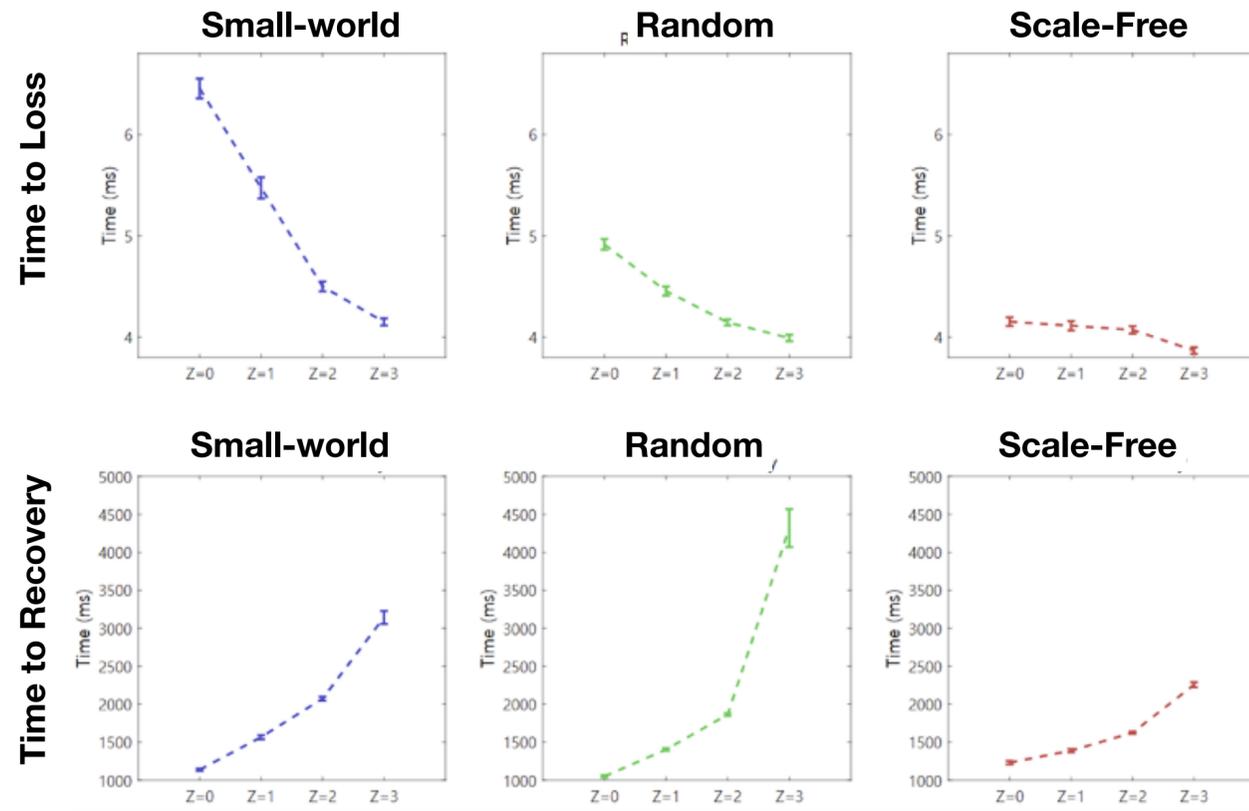


Figure S3. Relationships between ES Proximity (Adaptive Feedback Strength, Z), Network Topology, and Times to Critical State Loss and Recovery.

Diverse Stimulation Strength and Durations

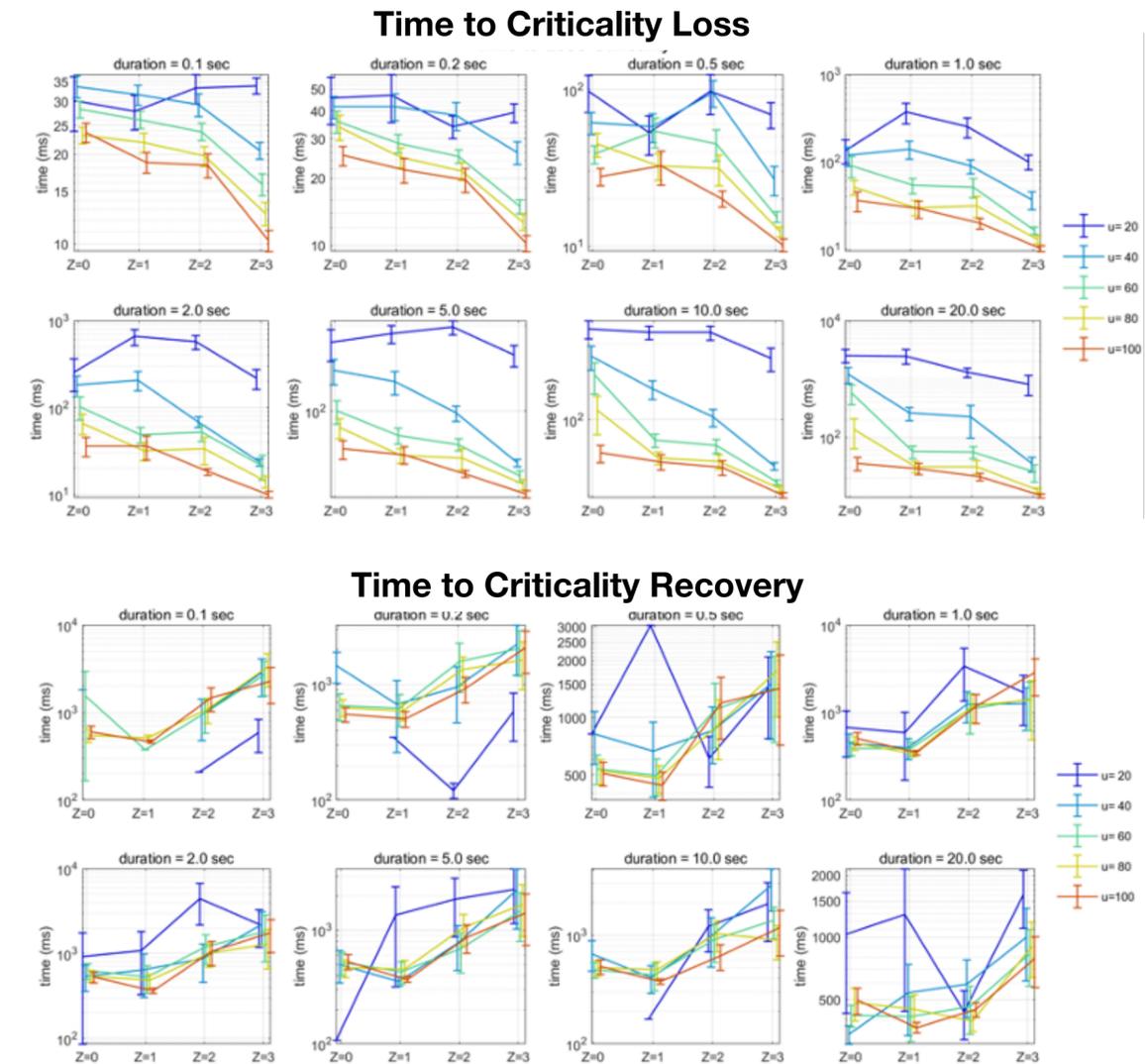


Figure S2. Relationships between ES proximity (adaptive feedback strength, Z), stimulation strength, stimulation duration, and the times to critical state loss and recovery. Networks with closer ES proximity (larger Z) exhibit shorter times to critical state loss and longer times to critical state recovery. This pattern is consistent across stimulation strengths ($u=40,60,80,100$) and stimulation durations (0.1, 0.2, 0.5, 1, 2, 5, 10, 20 seconds), except in cases where the stimulation is too weak ($u=20$) to deviate the baseline dynamics beyond three standard deviations. The relationship between ES proximity (Z) and the times to critical state loss and recovery becomes salient under external stimulations strong enough to push the network dynamics away from the critical state ($u>20$). Notably, the time scales of critical state loss and recovery depend on both the stimulation strength and duration.