Dept. of Physics

Asymmetry in Random Markov Models

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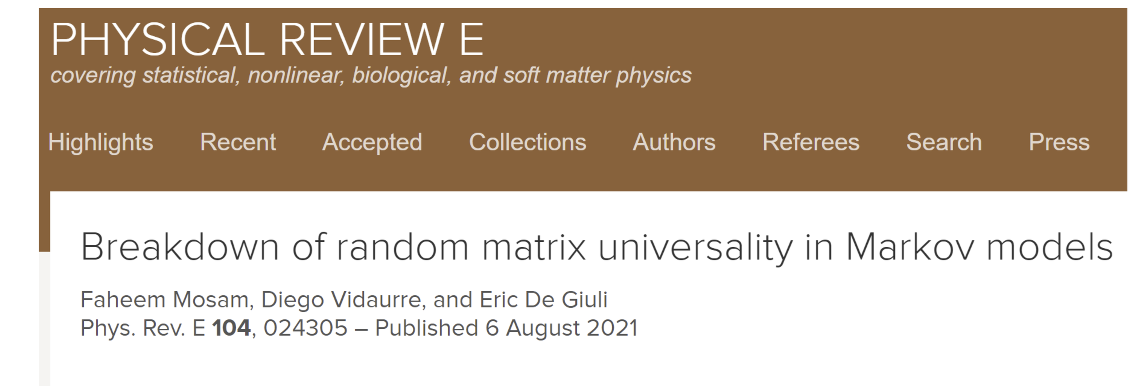


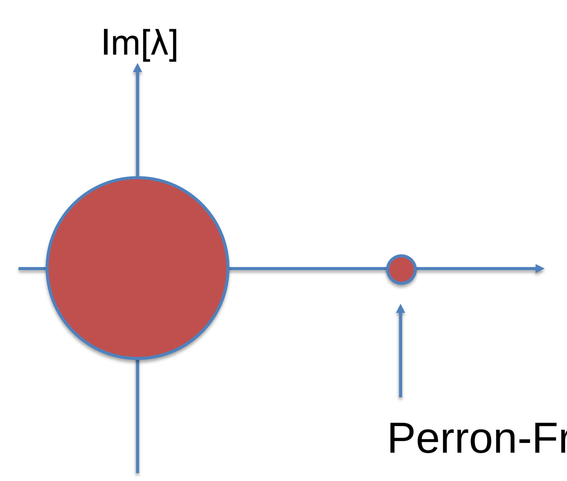
Summary

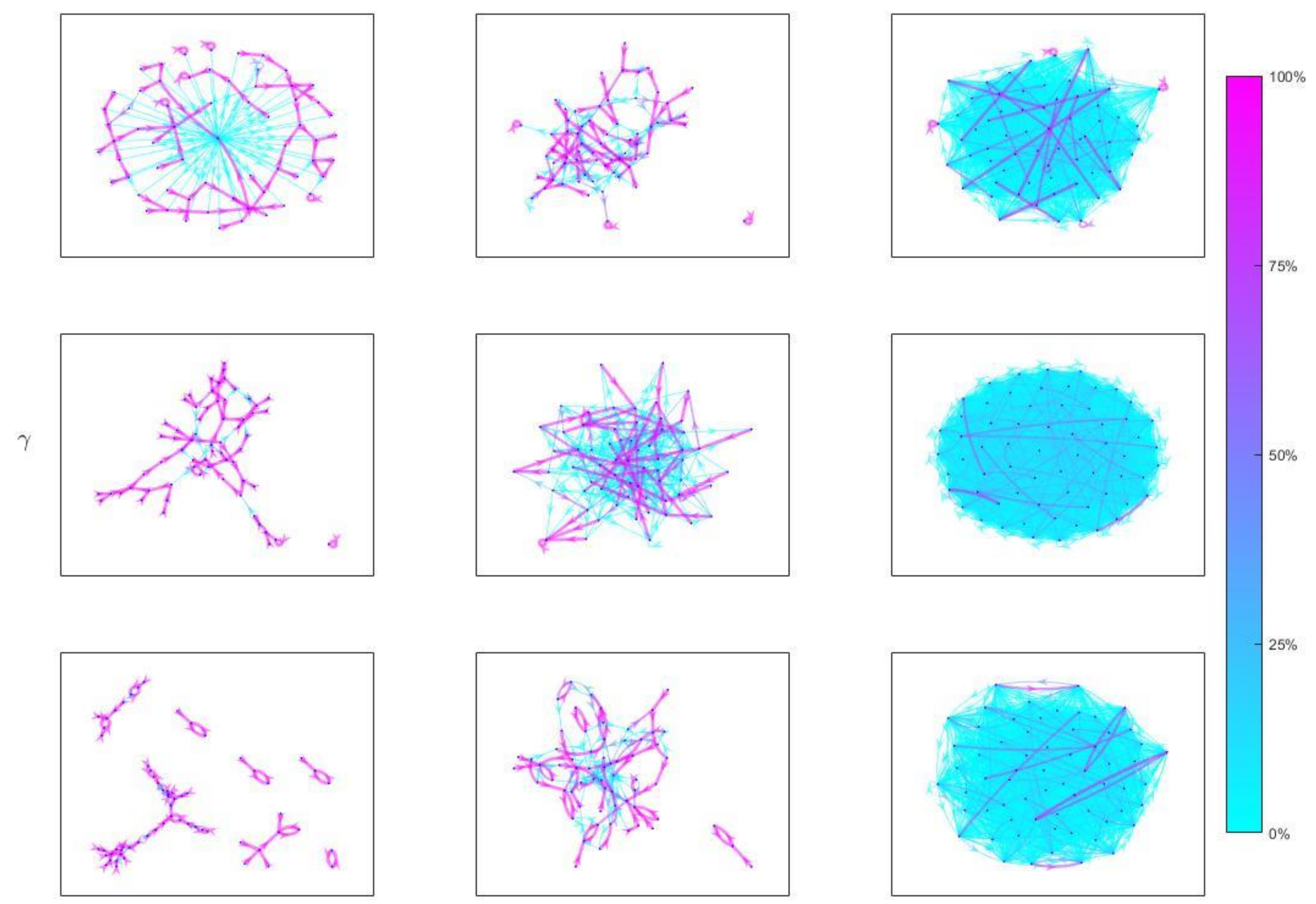
- Markovian systems are governed by adjacency matrices, allowing exploitation of Random Matrix Theory
- Define parameters to control the uniformity and correlation between paired states
- Model is able to produce rich results when analyzing both human brains at rest and monkey brains while undergoing anesthesia and waking up

1. Network representation & Information-theoretic quantities:

Write $M_{ab} = Q_{ab}/\sum_c Q_{ac}$

N → ∞ spectrum of Q: disk of radius $\lambda_c = \sigma/(\mu\sqrt{N})$.





What happens at finite N?

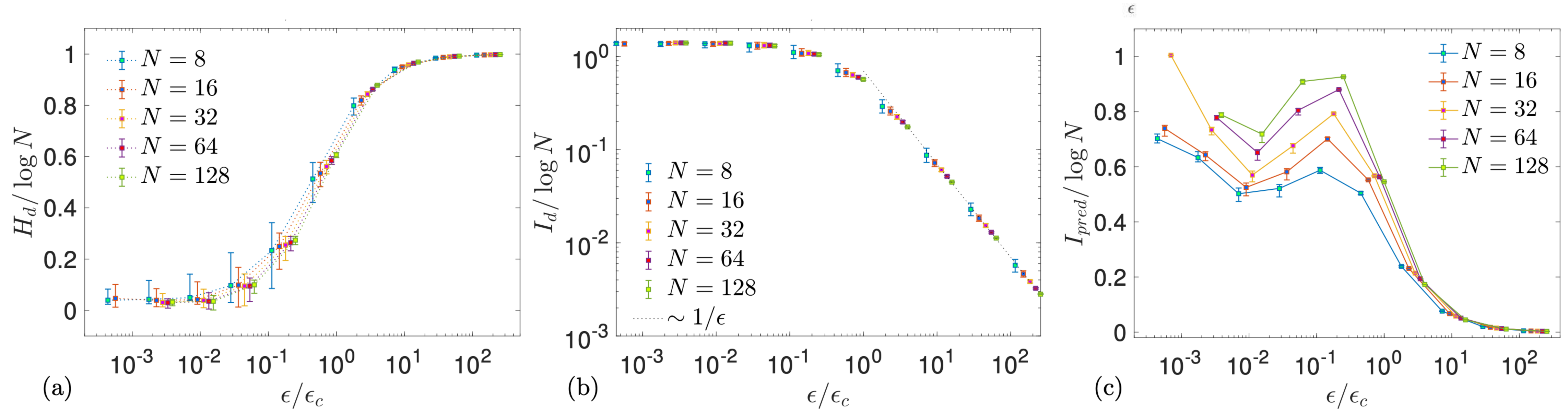


FIG. 4. Information-theoretic quantities. (a) The Shannon entropy rate of state visitation sequences in the Markov chain at indicated N; (b) Information rate, showing its increase as ϵ is decreased from large values; (c) Predictive information, peaking at an intermediate ϵ. Error bars in (a),(b) correspond to 20^{th} and 80^{th} percentile ranges over 1000 distinct Markov chains (samples) at each parameter value, while error bars in (c) correspond to true measurement errors from 300 distinct Markov chains (samples) at each parameter value.

- H_d measures information capacity [3], I_{pred} = I(past; future) measures complexity [4,6]

2. Neural fMRI data

- Hidden Markov Models previously determined for 820 human subjects [1]

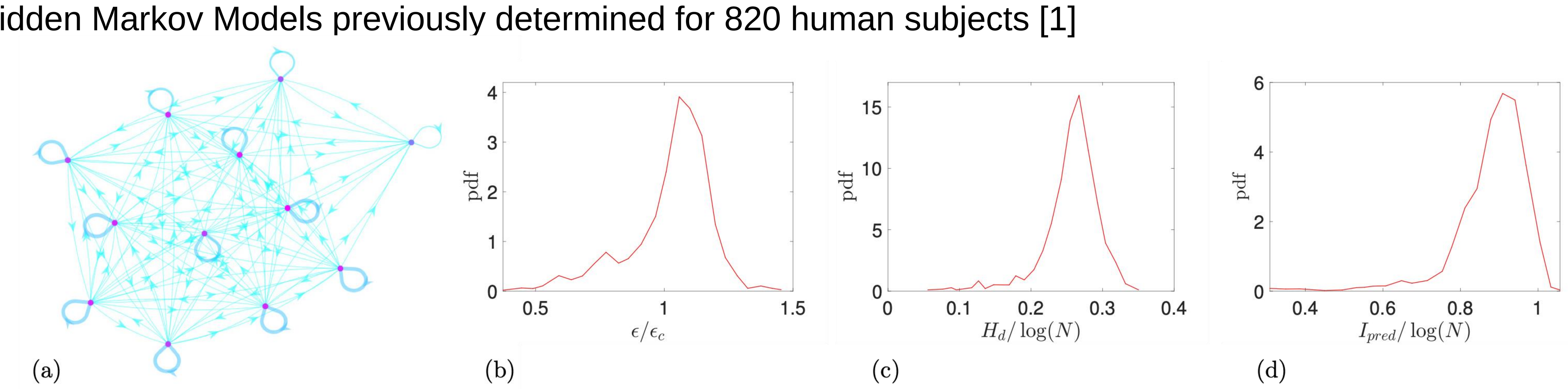
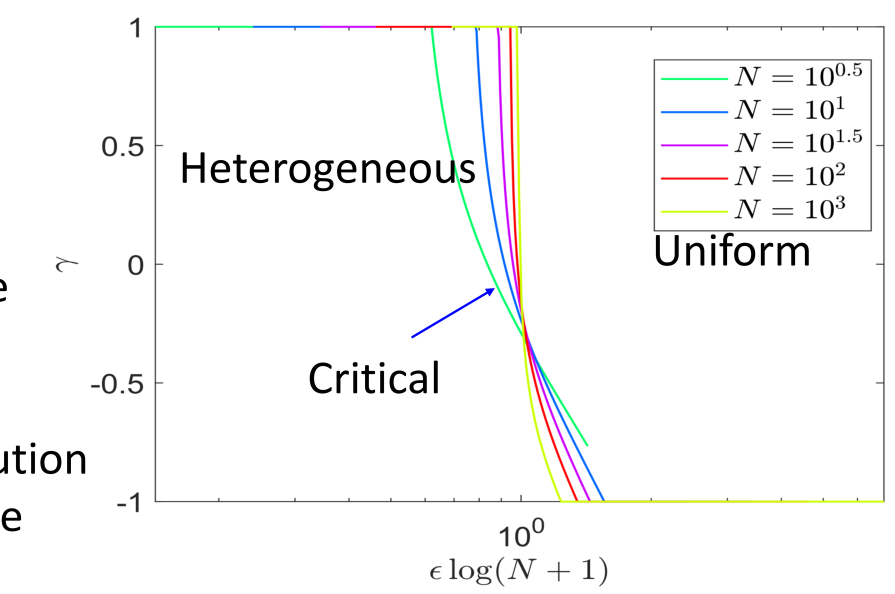
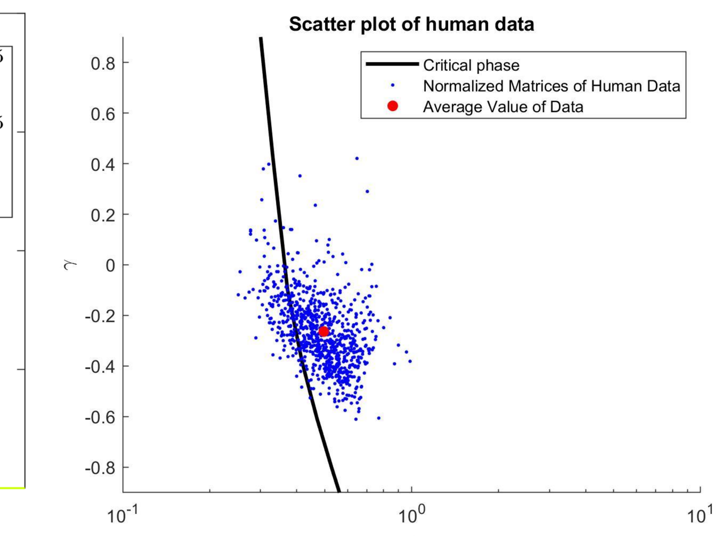


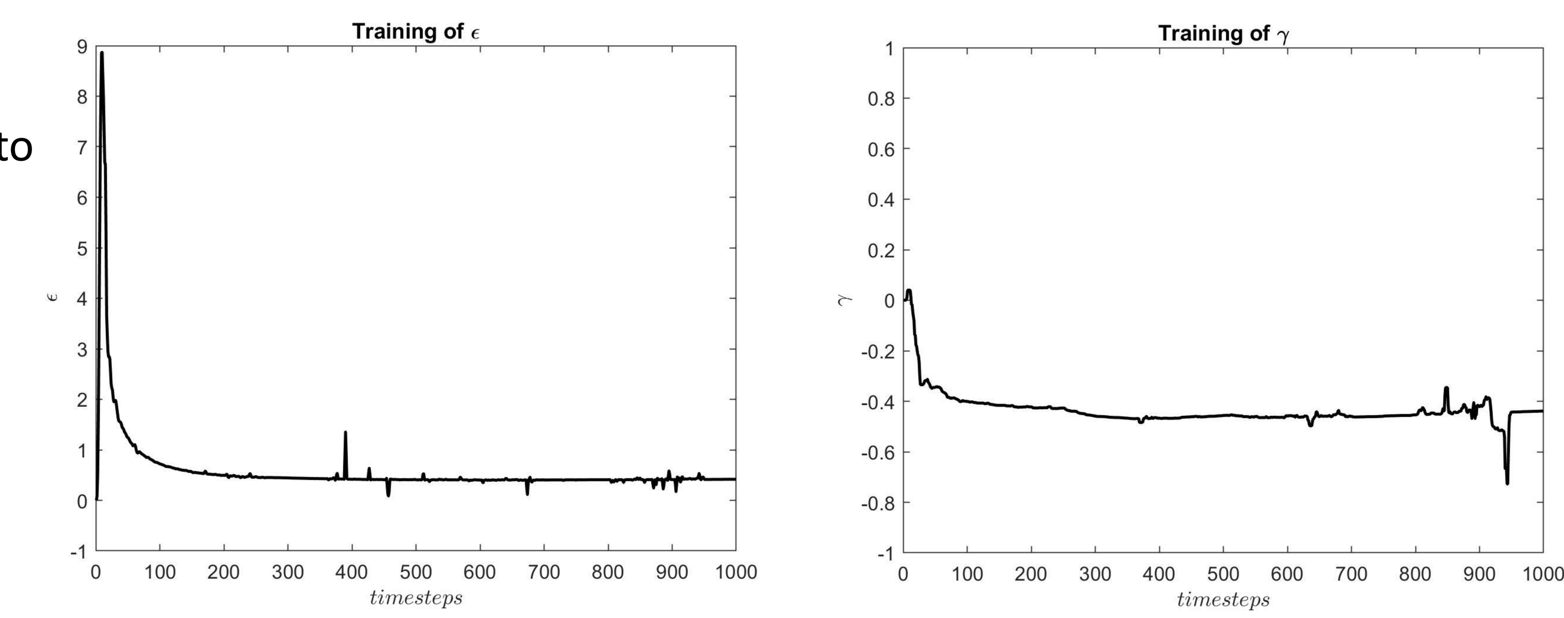
FIG. 8. Hidden Markov model applied to neural data. (a) Representative network; (b-d) Probability distributions of (b) temperature; (c) Shannon entropy rate of hidden sequences; (d) predictive information of hidden sequences.

3. Asymmetry and Phase Space:

- Theoretical critical value of γ is determined analytically
- Produces the plot to the right, showing that regardless of system size, there is a critical curve where structure emerges
- Previous human data was then analyzed to see where the distribution lies with respect to criticality in the phase space
- Previous human data was also used for a training algorithm on ε and γ
- Shows that values quickly converge to their expected values and remain there
- key parameter ε is conjugate to magnitude of matrix fluctuations and γ is conjugate to correlation between paired states

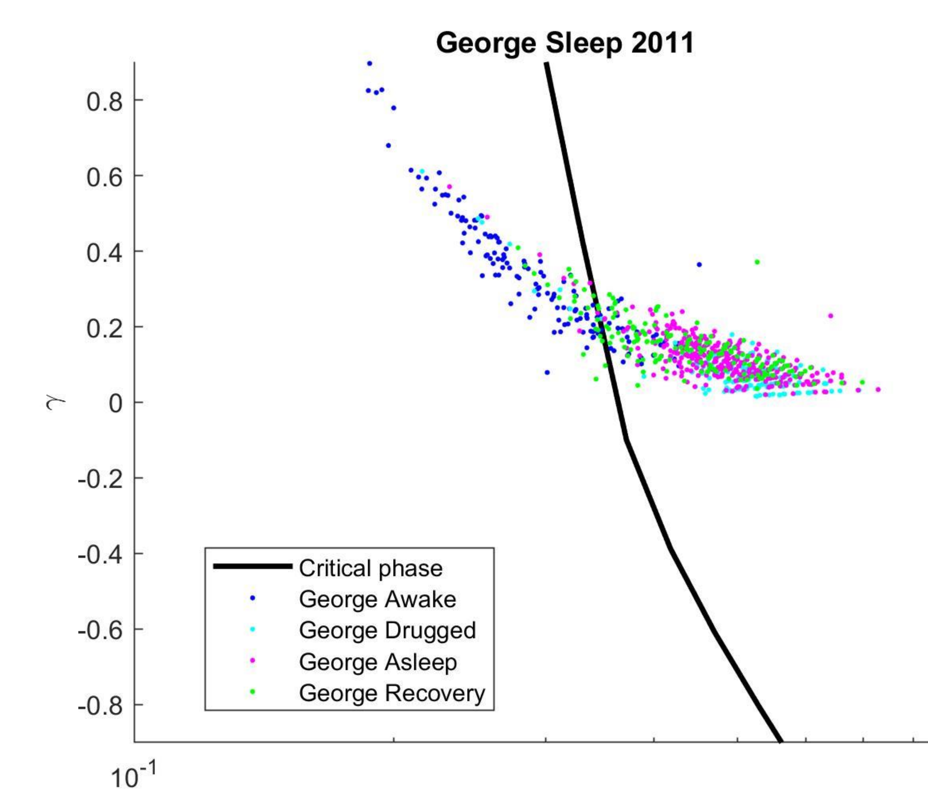






4. Observing ECoG data from monkeys

- 128 embedded nodes were used to develop Markov models at various partitions of time steps
- The full process models a monkey being awake, given anesthesia, then sleeping, then waking up
- Results show the monkeys brain undergoing a phase transition as it is drugged and undergoes the above process
- The phase space has potential to analyze further processes such as subjects at task or undergoing different activities
- Are there other processes that produce phase changes?



6. Conclusion

- Random Markov ensemble is a useful null model to understand emergence of complexity in Markov models
- key parameter ε is conjugate to magnitude of matrix fluctuations and γ is conjugate to correlation between paired states

References

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Acknowledgments

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