Multifractal scaling in the Landau-Ginzburg theory for cortical dynamics

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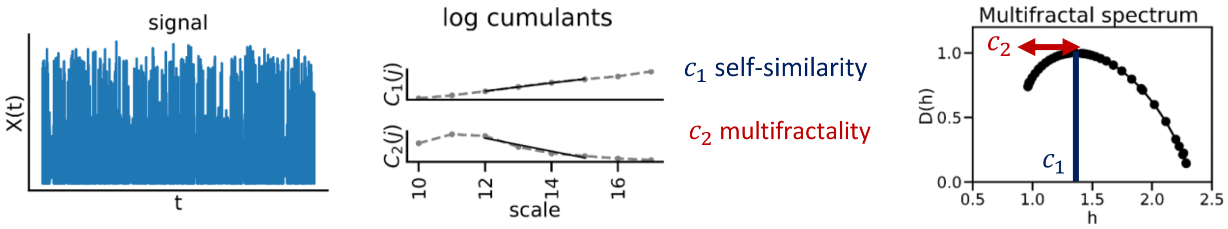
Introduction

Multifractal analysis (MFA) in the time domain investigates scale-invariance beyond self-similarity, in order to capture fluctuations in regularity

MFA has been applied to recordings of brain activity, but interpretation is difficult

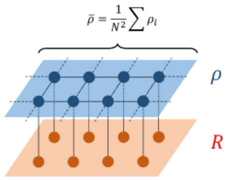
In non-invasive brain recording, spatial coarse graining at the sensor level requires *temporal* characterizations of criticality

We draw a link between multifractality and a model of brain criticality



Model

$$\begin{cases} \dot{\rho} = (R-a)\rho + b\rho^2 - \rho^3 + h + \sigma\sqrt{\rho}\,\eta + D\nabla^2\rho \\ \dot{R} = \dfrac{1}{\tau_R}(\xi - R) - \dfrac{1}{\tau_D}R\rho \end{cases}$$



- Derived from Wilson-Cowan model
  - 3rd order Taylor expansion of the excitatory activity
  - Timescale separation (inhibition >> excitation)
- Control parameters tune the model to criticality
  - τ_D controls ρ -> R coupling
  - ξ controls baseline resources

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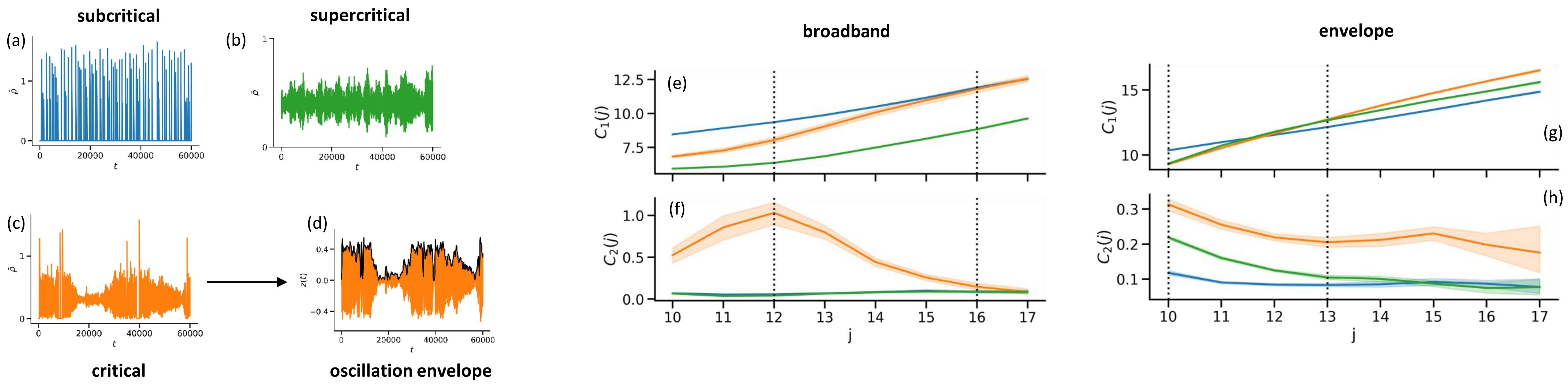
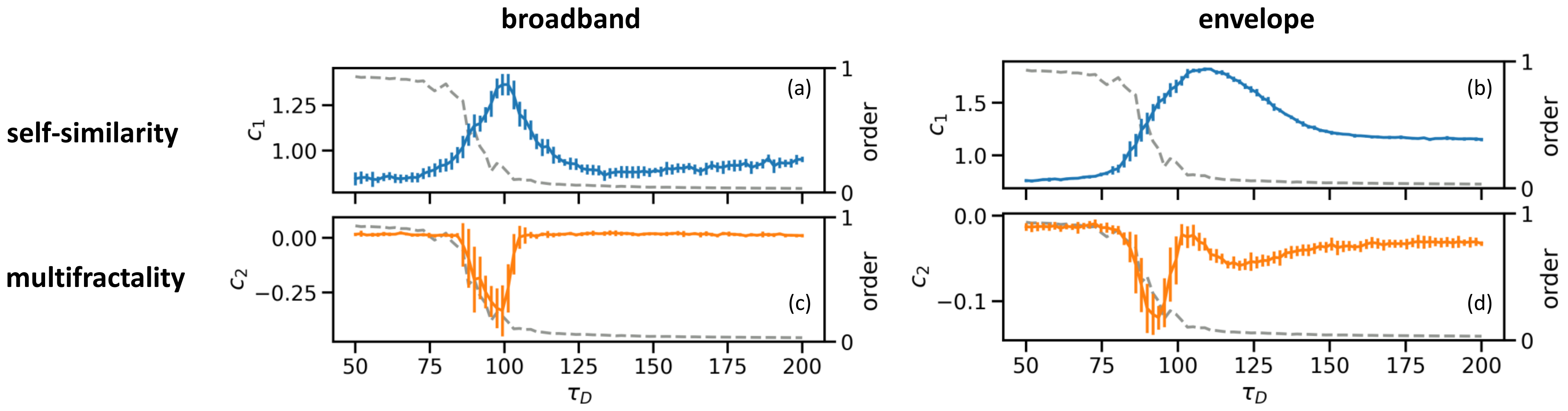


Mean field activity exhibits temporal scale-invariance

a-c) sample simulations of the mean field activity in the subcritical (blue), supercritical (green), and critical (orange) regimes.
d) amplitude envelope of the oscillation frequency band obtained via Morlet wavelet filtering.
Cumulant scaling functions for the broadband low-frequency time series e-f) and for the oscillation envelope g-h),
averaged over 20 independent simulations, shaded area indicates 90% CI. The characteristic temporal scale of oscillations is between scales 8 and 9.
The multifractal exponents c_1 and c_2 are estimated via linear regression over the range of scales delimited by the dotted lines

- Investigating the **scale-invariance** of the mean-field activity in the Landau-Ginzburg model, necessary for meaningful MFA.
- Observing the cumulant scaling functions $C_m(j)$ in the temporal scales larger than the oscillations, we look for **linearity** in a scale range.
- We compare two approaches from the literature **: broadband low frequency**, and the **amplitude envelope** of the simulated oscillations.
- The scaling range differs in the envelope: *scale-invariance is restricted to shorter time scales*.



Temporal multifractality outlines phase transition

Solid lines: exponents c_1 and c_2 with 90% CI error bars derived from 20 independent simulations, for the broadband signal a-b); oscillation amplitude envelope c-d).
Dashed lines: model order (Kuramoto synchronisation parameter).

- Comparing the multifractal exponents c_m with the model order, we find that they take **extremal values near the critical point**
- In the broadband, the self-similarity exponent c_1 takes extremal values near the true critical point; whereas the multifractal exponent c_2 takes extremal values over the extended phase transition.
- In the amplitude envelope, c_2 is extremal in the phase transition, but also exhibits a local minimum in the subcritical phase. c_1 takes its extremal value in the supercritical regime.
- In the critical regime, **both higher self-similarity and higher multifractality are observed**, consistent with previous studies of multifractality in human brain recordings.